Evaluation the Defects of Models Proposed for Social Networks

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ABSTRACT

In order to analyze the social networks, several models have been proposed and studied in the literature. Toivonen et al. [1] made a comparison between some of these models using the last.fm and email network datasets. Since these datasets were not comprehensive enough, some defects of the presented models were not appeared in their comparison. In this paper, we will try to achieve a better comparison using the collected datasets from the facebook and Wiki-Vote networks.

KEYWORDS

social network analysis, model, degree distribution, average neighbors degree, geodesic path length

1 INTRODUCTION

Small World (small average geodesic path length and large clustering coefficient), power-law degree distribution, inverse relation between clustering coefficient and vertex degree, and being assortative are the most important properties for most of social networks. These properties are known as "*signature of social networks*". Most of analyzed social network graphs have these properties. Evolution of presented models show that the researchers in this field have tried to present some models to simulate the mentioned properties.

Social network analysis has some basic elements include: Average degree, clustering coefficient, geodesic path length and assortativity. Average degree in an indirected graph is defined as:

$$\langle k \rangle = \frac{2 \times \#Number\ of\ links}{\#Number\ of\ vertices}. \quad (1)$$

Clustering coefficient of a graph is defined by Wasserman [2] as average clustering coefficient of vertices, and for each vertex clustering coefficient can be defined as:

$$C(G) = \frac{3 \times\ \#Number\ of\ triangles}{\#Number\ of\ connected\ triples\ in\ graph}. \quad (2)$$

Geodesic path length between a couple of vertices is shortest distance between that pair of vertices in graph, and the assortativity coefficient is the Pearson correlation coefficient of degree between pairs of linked nodes. In a assortative graph vertices with large degree connect to other vertices with large degree. If average number of neighbors degree increase with degree of vertices, graph is assortative.

The underpinnings of social network evolution are to be found in modeling and studying evolving graphs. Informally, evolution refers to a change that manifests itself across the time axis [3]. For understanding dynamics of social networks we need to model them. Designing a realistic model, which can describe mechanism of social network evolution is one of the challenging task in context of social network.

In 2009, Toivonen et al., [1] have compared the proposed models for social networks with real data. In their comparison, they used datasets collected from last.fm website and email network. These two datasets have fairly small clustering coefficient(between 0.2 and 0.3) and small average degree (fewer than 10).

Choosing these data had two problems. First, the social networks, from which these data were collected, are not comprehensive, i.e. these social networks have special users with special behavior. The First problem causes the second problem. Comprehensive social

networks have large clustering coefficient and large average degree. For example three comprehensive social network datasets were collected from facebook, twitter, and google plus have clustering coefficients, more than 0.5, and average degree more than 43 [4].
In this paper first, the web site used for collecting real datasets are introduced . then, the proposed models are explained briefly and finally the modes are compared.

2 REAL DATA

Our used datasets (facebook and Wiki-Vote) is taken from [4]. Facebook is a comprehensive social network with large clustering coefficient, and Wiki-Vote has small clustering coefficient. Clustering coefficient of Wiki-Vote was smaller than last.fm and email network, but it has still small world property. This gap capable us to reveal the models defects. Defects Which were not transpired in Toivonen's comparison.
In facebook graph nodes correspond to facebook users and edges are the friendship relations between these users. Wiki-Vote graph is a subgraph of wikipedia network. In this graph nodes show wikipedia users and edges is made by a voting system. A user vote to other user if he/she like that user's article, and by this way an edge emerges between these two users. These interactions make Wiki-Vote's social network. We will describe these graphs in next subsection.

2.1 real data structure

Facebook is an online general social network with more than one billion users. Each user in facebook is considered as a vertex, and friendship relation of two users is considered as link connecting their coresponding vertices. Date for Wiki-Vote has been collected from wikipedia forum. Some users produce contents in forum and some other users vote to his contents and some users with more votes are chosen as forum administrator. Nodes in the network represent wikipedia users and a directed edge from node i to node j represents that user i has voted on user j.
Both of datasets naturally have directed links. Since in the modeling existance of relations is important, not the nature of relations, we consider these graphs as indirected networks. And we will think about these directed links as interaction between two users.

3 CATEGORIZING AND INTRODUCING OF MODELS

3.1 Categorizing

In this section we will utilize from categories introduced by Toivonen et al. [1] for grouping social network models. Toivonen et al. divided social network models to two main groups: network evolution models (NEMs), and nodal attribute models (NAMs), and they also divided NEMs to two subgroups: dynamical network evolution models (dynamical NEMs), and growing network evolution models (growing NEMs). They showed each of these groups appear different behaviors.
Network evolution models have three primitive characteristics:

1. They change graph structure by repeating a special process on graph with adding and removing links and vertices.
2. In each time step they choose a subset of links or vertices, and remove some of these links or vertices by stochastic rules.
3. Each subgroups of NEMs has its own termination criterion. Growing NEMs stop when they reach to a determined number of vertices, and dynamical NEMs stop when their graphs reach to a stable statistical status.

Growing network evolution models usually start with an empty graph, and in each time step, add a new vertex to the graph, or draw some links between existing vertices. They iterate these process to reach a predetermined number of vertices.
Dynamical network evolution models start

Table 1. Category of social network models

group	Model
Dynamical NEMs	DEB MVS KOSKK
Growing NEMs	Vaz TOSHK
NAMs	BPDA WPR

with a graph which has predetermined number of vertices, and in each time step remove a number of vertices (and immediately add the same number of vertices) or links, and also add a number of links. Models iterate these to reach a stable statistical status.
Nodal attribute models, add linke between each pair of nodes, i.e. i and j, based on characteristics of vertices i and j.
Table 1 shows category of social network models. In subsequent subsection we introduce these models.

3.2 Algorithm of DEB

DEB was the first algorithm which introduced by Davidsen et al [5] in 2002. They would like to produce graphs with small world behavior. Algorithm 1 shows how this model makes graphs. DEB has just one free parameter i.e. probability of remove (p).

```
initialization: Make a graph with N
isolated vertices;
step := 0;
while Graph is not in a stable status do
    step := step + 1;
    choose randomly a vertex named i;
    if i has fewer than 2 neighbors then
        choose another vertex randomly
        named j;
        draw a link between i and j;
    else
        choose two neighbors of i named j
        and k;
        draw a link between j and k;
    end
    choose randomly a vertex r with
    probability p and remove r with all of
    links;
end
```

Algorithm 1: DEB algorithm

3.3 Algorithm of Vaz

In 2003, Vazquez presented a couple of models which produce graphs with local rules [6]. Only one of those models, which called *Connecting nearest-neighbor model* have social network signature in produced graphs. In this model there is only one free parameter u, which is the probability of adding new vertex. Algorithm 2 shows how Vaz model works.

```
initialization: make graph with only one
vertex;
step := 0;
NumberOfNodes := 1;
while NumberOfNodes ≤ N do
    if RandomNumber ≤ u then
        add a new vertex i;
        NumberOfNodes := NumberOfNodes + 1;
        draw a link between i and another
        randomly chosen vertex j;
        create potential link between i and
        neighbors of j these links are not
        real links;
    else
        change a potential link to real link
    end
end
```

Algorithm 2: Vaz algorithm

3.4 Algorithm of BPDA

Boguna et al. proposed BPDA model in 2003 [7]. Producing a graph with high cluster coefficient, degree-degree correlation, and community structure [1], by homophyly principle was their goals.
Homophyly principle at first established by McPherson et al. [10]. Essence of this principle summed up in this sentence: "birds of a feather flock together". Homophyly principle tells similar persons will be friend with more probability.
In this model, vertices at first scattered in one-dimensional segment $[0, h_{max}]$ and based on their Euclidean distance link drawn between them. Algorithm 3 showes that this model has two free parameters α and b. Parameter α is

[1] Communities are sub graphs of a graph which are more inter community link than intra community links for more read refer to [9]

homophyly coefficient and b is used for fitting average degree of graphs. We should choose such a big number as h_{max} which probability of vertices overlapping be very small, and increasing h_{max}, could not affect on resulting graph.

```
initialization: Scattered uniformly
vertices in one-dimensional segment [0, h_max];
d is Euclidean distance brtween i and j;
forall the pair of nodes do
    draw link between i and j with
    probability p = 1/(1+(d/b))^α
end
```

Algorithm 3: Algorithm of BPDA

3.5 Algorithm of MVS

This model was proposed in 2004 by Marsili et al. [8]. They tried to find an unstable state in graphs which produced by this model. This model has 3 free parameters: η probability of general search, ξ probability of local search, and λ probability of remove. Marsili et al. showed in a special range of ξ/λ, average degree and clustering coefficient of the model have unusual behaviors. Algorithm 4 is the algorithm of MVS.

```
initialization: make a graph with N
vertices;
step := 0 ;
while Graph is not in a stable status do
    step := step + 1;
    forall the vertices do
        choose randomly another vertex j and
        draw a link between current vertex
        and j with probability η ;
        with probability ξ draw a link
        between current vertex and one of
        the friends of friends which is
        selected randomly;
    end
    delete each of the links with
    probability λ;
end
```

Algorithm 4: Algorithm of MVS

3.6 Algorithm of TOSHK

Toivonen et al. in 2006 presented a model konwn as TOSHK [11]. TOSHK uses local rules for producing graphs. To make a graph with social networks signature as well as more realistic communities Toivonen et al. presented this model.

In principle, this model has three free parameters. One of them is the number of selected vertices for local search i.e. m_r. This parameter determined before starting the algorithm. Two other parameters are m_s, number of secondary search and p, probability of choosing one of the numbers $1 \dots m_r$ (Algorithm 5).

```
initialization: make a graph with one
vertex;
NumberOfNodes :=1;
while NumberOfNodes ≤ N do
    add node j to the graph;
    NumberOfNodes := NumberOfNodes + 1;
    choose 1 ≤ i ≤ m_r with probability p_i ;
    choose i vertices of graph randomly;
    n_2nd ~ U[1,k], k = 1,...,m_s;
    choose n_2nd number of neighbors of m_r
    chosen vertices;
    draw links between current vertex and
    these vertices;
end
```

Algorithm 5: Algorithm of TOSHK

3.7 Algorithm of WPR

Wong et al. in 2006 presented another model based on homophyly principle [12], which is known as WPR. There is some differences between BPDA and WPR. WPR distribute vertices in 2-dimensional space unlike BPDA, that scattered vertices uniformly in 1-dimensional space. WPR scattered vertices according to a homogeneous Poisson point process on space (see [13] [14]).

There is four free parameters in this model: H, p, p_b, p_Δ. Parameter H is the neighbourhood radius, p is the average density of the network, p_b is the proximity bias which specifies the sensitivity to geographical space by the actors on establishing social links, and p_Δ maintain the expected average density of graph. WPR is given in Algorithm 6.

3.8 Algorithm of KOSKK

Kumpula et al. in 2007 proposed KOSKK in [15]. This model was proposed to model

```
initialization: scatter vertices in a
2-dimensional space according to a
homogeneous Poisson point process;
forall the pair of vertices do
    if Euclidean distance is less than H then
        draw link between current vertices
        with probability p+p_b;
    else
        draw link between current vertices
        with probability p−p_Δ;
    end
end
```

Algorithm 6: Algorithm WPR

graphs with weighted links. Some of the network characteristics, in addition to interactions, depends on weights of these interactions. KOSKK draw new links according to the weights of links incident to a vertex. This model has tree free parameters. Parameter p_Δ is probability of local search, p_g is probability of general search and p_r is probability of removing the node (Algorithm 7).

4 MODEL ANALYSING

In this section we will analyze graphs which produced by the models. But before, we should fit parameters to produce graphs, which have same value for some properties with real data. After this fitting, we compare other properties of graphs.

4.1 Fitting the models

Priority of properties for fitting models in [1] are: average degree of the graph, clustering coefficient, assortivity and average geodesic path. In this paper we will use this priority.

As mentioned DEB and Vaz have only one free parameter so we just can fit this parameter to simulate first property i.e. average degree. This does not mean that other property could not be matched with real data, but means that we have not any tools to control other properties.

Some of the other models have two or three parameters, which can control clustering coefficient in addition to average degree. BPDA, MVS and TOSHK are in this group of models.

```
initialization: make a graph with N
isolate node;
set δ and W_0;
while Graph is not in a stable status do
    forall the vertices i do
        if current vertex has at least one
        neighbor then
            choose one of i's neighbors j
            according weigth of j which is
            the summation of the weigth of
            the incident links;
            add δ to the weight of link {i,j};
            choose one of the j's neighbors
            k according weigth of k;
            add δ to weight of link {j,k};
            if there is a link between i and k
            then
                add δ to the weight of link
                {i,k};
            else
                with probability p_Δ draw a
                link between i and k, set the
                weight {i,k} weight W_0;
            end
        end
        if current vertex has at most one
        vertex, or with probability p_g then
            choose randomly another vertex j;
            if there is a link between i and j
            then
                add δ to the weight of link
                {i,j};
            else
                draw a link between i and j
                and set the weight of link
                {i,j} weight W_0;
            end
        end
    end
end
```

Algorithm 7: Algorithm of KOSKK

KOSKK and WPR have some other free parameters, that can be used to control some other properties. KOSKK has δ which can be fitted to simulate assortative and community structure properties. This parameter varies between 0 and 1. When it approaches to 0, assortativity coefficient increase, but community structure is destroyed in resulting graphs. When δ approaches to 1, assortativity coefficient decreases and community structure is made. Community structure is one of the most important feature of social networks. When δ approaches to 1 assortativity coefficient decreases but graph still remained assortative.
Toivonen et al. [1] set $\delta = 0.5$ and with this value for δ, they obtained very small measure for assortative coefficient. Notwithstanding we wont analyze community structure in this paper but to produce graphs with community structure (our real data are so clustered) we increase δ to 0.85. With this value for δ, our resulting graphs are assortative. For WPR we fit parameter to produce graphs with average geodesic path length similar to real data. Table 2 showes fitted model parameters.
Results are averaged over 20 realizations of each network model and come on Tables 3 and 4. In the following section we take a closer look at this results. We produce graphs with $\langle k \rangle = 43.69 \pm 0.5$ and $\langle k \rangle = 29.14 \pm 0.5$ to model facebook and Wiki-Vote graphs respectively.

4.2 DEB analyzing

As we mentioned in the preceding section, this model has only one free parameter i.e. probability of remove, and for this reason, we do not have any control on clustering coefficient. We can see this result in Tables 3 and 4. Clustering coefficients of this model are not matched with real data. This problem also exist in Toivonen et al [1] results.
Initial purpose of this model was producing small-world graphs. So resulting graph should have small average geodesic path length. This is obvious in Tables 3 and 4. This model's average geodesic path length is even smaller than the real data.

4.3 Vaz analyzing

Vaz like DEB has just one free parameter, so (and again like DEB) we only can control average degree of produced graphs. Tables 3 and 4 show, properties of this model. Average geodesic path length of this model is small. For facebook graph this model graphs geodesic path length is smaller, but for Wiki-Vote this property is bigger than real data. With up together these results and Toivonenet et al. [1] results, we can conclude that for graphs with small average degree, diameter of graph is large but for dense graph, the diameter becomes smaller.

4.4 BPDA analyzing

This model and the following models have some defects in modeling of social networks which were not appear in Toivonen et al. comparison. In the remaining models, we have more than one free parameters, so we should control some other properties, in addition to average degree.
Cliques (fully connected subgraphs) are one of the most important features in social networks. Among all of the cilque types, triangle is the most important. Being friend with friends of friends is common relations in social networks and providing models, for simulating this feature of real data is one of the most important task of researchers.
Before revolution of online social networks like facebook, twitter and google plus online social networks have small clustering coefficient. But These days dense social networks with large clustering coefficient have emerged (see collected data in [4]). For this purpose we want to show how these models are not efficient enough, and why we need new models for modeling new social networks.
BPDA was first nodal attribute model which proposed. This model in Toivonenet et al. [1] comparison produced graph which its clustering coefficient were almost matched with real data's clustering coefficients. For graphs which model's parameters fitted to facebook graph this model, unlike the DEB

Table 2. Targeted network features, and the fitted model parameters

Name	Real data	Parameters
DEB	facebook	$p = 0.011$
	Wiki-Vote	$p = 0.016$
Vaz	facebook	$u = 0.966$
	Wiki-Vote	$u = 0.945$
BPDA	facebook	$\beta = 44,\ \alpha = 3,\ d = [1, 11000]$
	Wiki-Vote	$\beta = 320,\ \alpha = 1,\ d = [1, 70000]$
MVS	facebook	$\eta = 0.0005,\ \xi = 0.0934,\ \lambda = 0.004$
	Wiki-Vote	$\beta = 0.0015,\ \xi = 0.06640,\ \lambda = 0.0043$
TOSHK	facebook	$p_1 = 0.999,\ p_2 = 0.001,\ k = 189$
	Wiki-Vote	$p_1 = 0.01,\ p_2 = 0.01,\ p_3 = 0.75,\ p_4 = 0.23,\ k = 7$
WPR	facebook	$p = 0.07018,\ p_b = 0.57,\ p_\Delta = 0.06948,\ H = 2$
	Wiki-Vote	$p = 0.0685,\ p_b = 0.08219,\ p_\Delta = 0.068,\ H = 2$
KOSKK	facebook	$p_\Delta = 0.11,\ p_g = 10^{-6},\ p_r = 0.001$
	Wiki-Vote	$p_\Delta = 0.09,\ p_g = 0.06,\ p_r = 0.002$

Table 3. Basic statistics of the facebook sets and the models fitted to each. Results are averaged over 20 realizations of each network model

Name	Number of nodes	Average degree	Clustering coefficient	Redious	Diameter	Shortest path length
Facebook	4039	43.69	0.6	5	8	3.93
DEB	4039	43.9	0.38	4	6.3	2.78
Vaz	4039	43.52	0.34	4.7	8.8	3.66
BPDA	4039	43.19	0.61	39.5	78.6	26.76
MVS	4039	43.98	0.035	3	4	2.607
TOSHK	4039	43.97	0.35	4	7.3	2.97
WPR	4039	43.56	0.59	4	5.5	3.06
KOSKK	4039	43.83	0.22	1.3	7.4	2.73

and Vaz, produced graphs, that their clustering coefficient matches with facebook graph's clustering coefficient (Table 3). It seems that this model produces graphs with a wide range of clustering coefficients, but this is not true, as you can see in Table 4, clustering coefficient of graphs fitted to Wiki-Vote is larger than Wiki-Vote's graph clustering coefficients. We tried to produce graphs, whose clustering coefficient match with Wiki-Vote but, we failed .

In this model we have two free parameters α and b. Parameter b is used for matching average degree, thus, we have α for controlling clustering coefficient of produced graph. As you can see in Table 2 for produced graphs, similar to Wiki-Vote graph we choose $\alpha = 1$ i.e. smallest value. Even with this parameter we could not produce graphs with clustering coefficient smaller than 0.154 (Table 4). We can claim, there is a lower bound for BPDA's clustering coefficient spectra.

Having lower bound for clustering coefficient spectra is not the only problem of this model. As you can see in Tables 3 and 4, radius , diameter and average geodesic path length of this model is considerably more than real data. This has been shown in Toivonen comparison too. In facebook graph three vertices with most degrees have 1045, 792 and 755 neighbors, but in BPDA's graphs (averaged over 20 realizations), vertices with most degrees have only 66, 64.8 and 63.5 neighbors. We know that hubs make short cut between vertices and decrease their smallest path length [16] [17]. Random graphs has vertices with small degree and as Wang and Chen have shown in [16] random graphs have somewhat larger

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Table 4. Basic statistics of the Wiki-Vote sets and the models fitted to each. Results are averaged over 20 realizations of each network model

Name	Number of nodes	Average degree	Clustering coefficient	Redious	Diameter	Shortest path length
Wiki-Vote	7115	29.14	0.14	1	7	3.24
DEB	7115	29.01	0.36	3.8	7.4	3.15
Vaz	7115	28.78	0.32	5.4	10.4	4.02
BPDA	7115	29.05	0.154	14	28	10.04
MVS	7115	29.24	0.04	4	4	2.94
TOSHK	7115	28.79	0.14	3	5.3	2.95
WPR	7115	28.9	0.14	4	5.6	3.31
KOSKK	7115	29.43	0.14	3.4	6.3	3.07

average geodesic path length, than scale-free networks, and scale-free means emergence of hubs. In following, we show this model has other similarities to random graphs.

4.5 MVS analyzing

This model uses from link deletion. Toivonen et al [1] claim that for small average degree of graph this model could not produce large number of cliques, this is true according to the theoretical and numerical analysis in [8]. But there is another problem, when average degree becomes larger, after a threshold , the larger average degree means the smaller clustering coefficients. Asymptotically clustering coefficient for large average degree convergence to a very small number (maybe less than 0.01).
Our graphs have large graph average degree, so we expect clustering coefficient to be very small. This is true according to numerical results in Tables 3 and 4. Toivonen et al [1] results for clustering coefficient was match to real data results because, their data had small average degree (under the threshold), and as we showed this cluster even could not have small world behavior (because its small clustering coefficients). MVS have small average geodesic path length even smaller than real data.

4.6 TOSHK analyzing

In this model to enlarge clustering coefficient we should increase p_1 in one side, and decrease p_2 in the other side. For example to produce two new links (for instance $k = 1$) if we use from p_1 we make two new triangles, but with p_2 we can not make any triangle. So we enlarged p_1 as far as we could, nevertheless clustering coefficient only reached to 0.35 for graphs similar to facebook real data.
Toivonen et al. [1] mentioned, TOSHK is unable to produce graphs with small clustering coefficients. This is true if we confine ourselves to implemented version of TOSHK in [11]. Author in implemented version [11] confined themselves to at most two general search when new vertex is added i.e. $m_r = 2$. But as it can be seen in Algorithm 5, TOSHK algorithm is so flexible. When we increase number of general searches we could reach to smaller clustering coefficient, and truly, this is what we did. We set $m_r = 4$ and reached to our purpose. For modeling Wiki-Vote by TOSHK we produce graph, which clustering coefficient were 0.14, equal with Wiki-Vote clustering coefficient.
As we can see in Tables 3 and 4, Average geodesic path length of this model is smaller than real data.

4.7 WPR analyzing

So far none of the discussed models could produce wide range of clustering coefficient spectra. But this model can produce graph with such feature. As it can be seen in Tables 3 and 4 this model produce graphs with large clustering coefficient (equal with real data). This model geodesic path, also is very small number and near to real data.

4.8 KOSKK analyzing

Modeling weighted graph was purpose of this model. This model like MVS after a threshold of average degree clustering start to diminish. For instance for small average degree (under 10) clustering coefficient arise up to 0.67 but for large average degree it becomes smaller. For this reason, Toivonen et al. had acceptable results. For our comparison when we try to model facebook graph our clustering coefficient reach to 0.22. We take general search probability a very small number (Table 2) but even this small number could not remedy our probelm.
For modeling Wiki-Vote, Which had small clustering coefficient, there is not any problem, this is obvious in Table 4. This model has small geodesic average path length.

5 PLOTS OF MODELS

As Mislove [18] and Ahn [19] mentioned, Social networks have special behavior for degree distribution, Clustering coefficient as a function of the vertex degree and Average nearest-neighbor degree as a function of the vertex degree. Social networks have power-law degree distribution. There is inverse relation between clustering coefficient and degree of vertices and for social networks average nearest-neighbor degree of a vertex increase by its degree. In this section we compare this behavior of models with real data.

5.1 Degree distribution

Barabasi and albert mentioned in [17], which with preferential attachment [2] power-law destribution emerges. This reflects in our models, All the models which have preferential attachment (except MVS) have tail between poisson distribtion and power-law distribution (Figures 1(b), 1(c), 1(f) and 1(h)) as mentioned by Tiovonen et al. [1]. NAMs models tails decay like poisson distribution (Figure 1(d) and 1(g)) i.e. faster than power-law distribution, absence of vertices with high degree prove this issue. Although MVS have preferential attachment as local search but his distribution is similar to poisson distribtion (Figure 1(e)) it seems link removing play an important role in degree distribution of MVS.

[2] preferential attachment known as rich become richer, pereferantial attachment says that the likehood of a node being attachment to new link is proportional to the node degree

5.2 Clustering spectrum

Clustering spectrum $C(k)$ of the Wiki-Vote data and models fitted to it is shown in Figure 2. As mentioned, social networks clustering coefficient have inverse relation with degree of verticies. All of NEMs have this behavior (Figures 2(b), 2(c), 2(e), 2(f) and 2(h)).
Toivonen et al. showed that for NAMs there is a constant relation between clustering coefficient and degree of verticies, this is true for BPDA (Figure 2(d)) but our result for WPR (2(g)) showes direct relation between clustering coefficient and degree of vertices. Maybe this difference comes from homogeneous Poisson point process. In our fitted models (for facebook and Wiki-Vote), poisson distribution mean and neighbourhood radius were 7.5 and 2 respectively (facebook fitted version of WPR has direct relation too which has not shown in Figure), but Tiovonen et al. [1] neighbourhood radius was less than 0.05. This maybe cause to this difference. Another probability is this, with increasing average degree of graph, direct relation arise. Because there is no theoretical proof of these claims in hand, so more numerical simulation needs to prove this claims.

5.3 Average neighbors degree

Average nearest-neighbor degree as a function of the vertex degree for facebook and models fitted to it, is shown in Figure 3. As you can see in Figures 3(b), 3(c), 3(d), 3(f), 3(g) and 3(h) except MVS which is almost disassortative, all of the models are assortative and have same behavior with real data. Toivonen et al. mentioned that this behavior of MVS return to his nature i.e adding and deletion of nodes.
It is good to recall, KOSKK could have more

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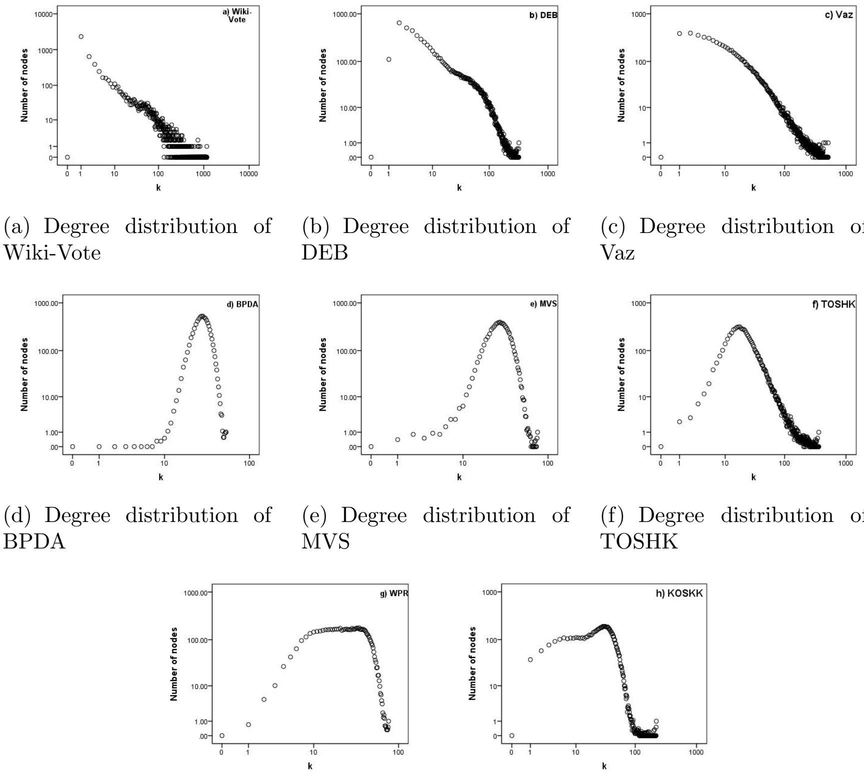


(a) Degree distribution of Wiki-Vote

(b) Degree distribution of DEB

(c) Degree distribution of Vaz

(d) Degree distribution of BPDA

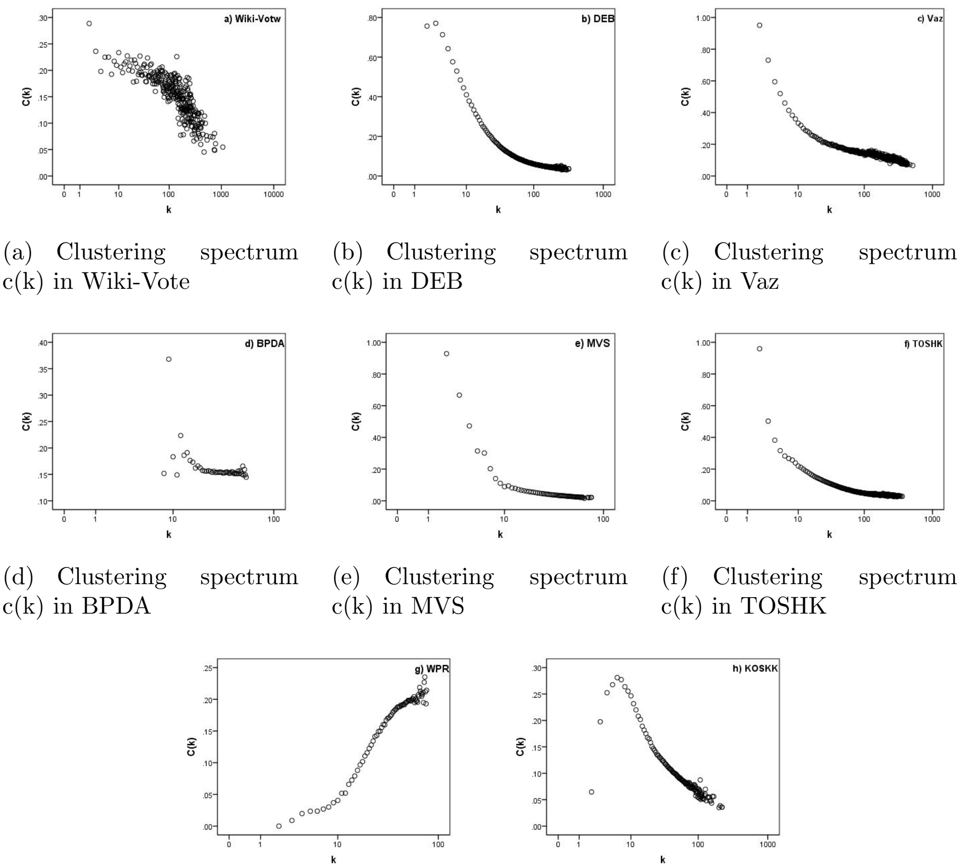
(e) Degree distribution of MVS

(f) Degree distribution of TOSHK

(g) Degree distribution of WPR

(h) Degree distribution of KOSKK

Figure 1. Degree distributions P(k) of the Wiki-Vote data and models fitted to it. Results are averaged over 20 realizations of each network model fitted to Wiki-Vote graph



(a) Clustering spectrum c(k) in Wiki-Vote

(b) Clustering spectrum c(k) in DEB

(c) Clustering spectrum c(k) in Vaz

(d) Clustering spectrum c(k) in BPDA

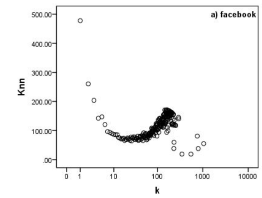
(e) Clustering spectrum c(k) in MVS

(f) Clustering spectrum c(k) in TOSHK

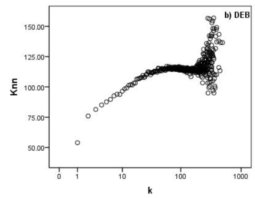
(g) Clustering spectrum c(k) in WPR

(h) Clustering spectrum c(k) in KOSKK

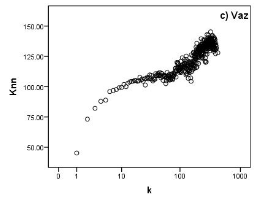
Figure 2. Clustering spectrum c(k) for the Wiki-Vote and data models fitted to it. Results are averaged over 20 realizations of each network model fitted to Wiki-Vote graph.



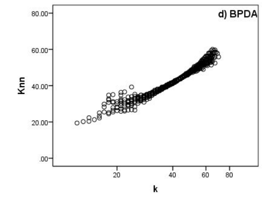
(a) Average neighbors degree Knn in Wiki-Vote



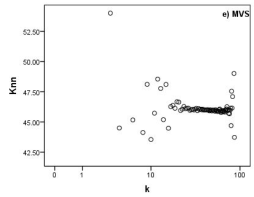
(b) Average neighbors degree Knn in DEB



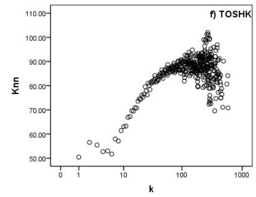
(c) Average neighbors degree Knn in Vaz



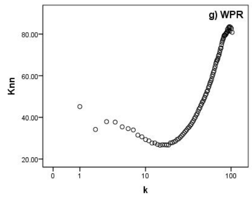
(d) Average neighbors degree Knn in BPDA



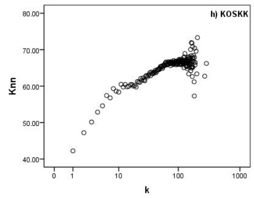
(e) Average neighbors degree Knn in MVS



(f) Average neighbors degree Knn in TOSHK



(g) Average neighbors degree Knn in WPR



(h) Average neighbors degree Knn in KOSKK

Figure 3. Average neighbors degree Knn for the facebook data and models fitted to it. Results are averaged over 20 realizations of each network model fitted to face book graph.

assortative coefficient but our choice of δ cause to have less steep.

6 CONCLUSION

As we showed, models which provided as yet could not produce all behavior and features, i.e. signature, of social networks. Some could not produce graph with large clustering coefficient and others do not have power-law degree distribution. Some have direct relation between clustering coefficient and degree of verticies and others are not assortative.
We had a condense and comprehensive glimpse on proposed models for social network, and reveal latent aspects of these model, which neglected in previous comparison. We showed that more work is needed in this area. Up together this result, show we need some new models, for modeling new social networks, networks which become denser and more clustered.

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